Saddle point states in 2D superconducting film biased near the depairing current

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The structure and energy of saddle point (SP) states in 2D superconducting film of finite width w with transport current I are found in the framework of Ginzburg-Landau model. We show that very near depairing current I_{dep} the SP state with a vortex does not exist and it transforms to 2D nucleus state, which is a finite region with partially suppressed order parameter. It is also shown that for slightly lower currents the contribution of the vortex core energy is important for SP state with a vortex and it cannot be neglected for $I \gtrsim 0.6 I_{dep}$. It is demonstrated that in the film with local current concentration near the bend the energy of SP state may be much smaller than one in the straight film and it favors the effect of fluctuations in such a samples.

I. INTRODUCTION

Many physical systems have several metastable states at fixed external parameters (temperature, magnetic field, etc.) which correspond to different local minima of their free energy. As an example it could be mentioned the different configuration of DNA molecule [1], vortex 'molecules' in the mesoscopic superconductors [2, 3], or magnetic states of the lattice of magnetic nanocaps [4]. The metastable states are usually separated from each other by the energy barrier ΔF which could be overcome due to thermal activation. If the height of the energy barrier is much larger than the thermal energy $k_B T$ then such a transitions are relatively rare events and one may estimate their rate as $\sim exp(-\Delta F/k_B T)$ [5]. The standard procedure to calculate ΔF is to find the saddle point states which correspond to the local maxima of the free energy and find the energy difference between them and the metastable states [1–3, 5].

In the superconductors with transport current less than critical one this idealogy could be also applied for calculation of the finite resistance appearing due to fluctuations. For 1D superconducting wires Langer and Ambegaokar (LA) [6] in framework of the Ginzburg-Landau (GL) model find that in the saddle point state the superconducting order parameter $\psi = |\psi|e^{i\varphi}$ is partially suppressed in the finite region along a wire and the amplitude of suppression depends on the current. If one starts from such a 1D nucleus state the time evolution of the order parameter will inevitably lead to the phase slip [6] and to the finite resistance R_{wire}. LA find dependence of ΔF both on the current and temperature which with good accuracy [7] could be written as

$$\frac{\Delta F_{LA}}{F_0} = \frac{2\sqrt{2}}{3\pi} \frac{w}{\xi} \left(1 - \frac{I}{I_{dep}}\right)^{5/4} \qquad (1)$$

where $F_0 = \Phi_0^2 d/16\pi^2\lambda^2$, Φ_0 is a magnetic flux quantum,

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w is a width and d is a thickness of the wire respectively, λ is a London penetration depth, ξ is a coherence length and $I_{dep} = c\Phi_0 wd/12\sqrt{3}\pi^2\xi\lambda^2$ is a depairing current in the Ginzburg-Landau model. According to the general concept $R_{wire} = \nu\, exp(-\Delta F_{LA}/k_B T)$ where the pre-exponential factor ν is calculated in Refs. [8, 9].

In contrast to 1D wire in thin $(d \ll \lambda)$ 2D film several saddle point states exist at given value of the current. Further we discuss the case of the relatively narrow film with $\xi \ll w < \lambda^2/d$ (this condition ensures the uniformity of the current distribution over the width of the superconductor with transport current in the ground state). At the present time it is distinguished three types of the saddle point states in such a samples: i) the state with a single vortex [10–15], ii) the vortex-antivortex (VA) state [14–17] and iii) LA state extended to 2D case [13–15]. In Ref. [15] it was argued that VA state has at least twice larger energy than the single vortex state and LA-like state is the most energetically unfavorable among the considered states at all currents $I \leq I_{dep}$. However in Ref. [15] the current dependence of ΔF_{LA} (see Eq. (1)) was not taken into account. If one takes the result for the energy of the single vortex (V) state near depairing current $1 - I/I_{dep} \ll 1$ (found in the London model [13–15])

$$\frac{\Delta F_V}{F_0} \simeq 1 - \frac{I}{I_{dep}} \qquad (2)$$

and compare it with Eq. (1) then it is easy to see that even for wide films $w \gg \xi$ there is a finite region of the currents $1 - (3\pi\xi/2\sqrt{2}w)^4 < I/I_{dep} \leq 1$ where $\Delta F_{LA} < \Delta F_V$. But such a quantitative comparison is not correct because Eq. (2) does not contain the energy of the vortex core ($E_{core} \sim 0.38 F_0$ for vortex located far from the edges [18]) and, as we show below, it seriously underestimates ΔF_V at $I \gtrsim 0.6 I_{dep}$.

The effect of the vortex core was taken into account in the recent work [13] using GL model. But authors focused on the region of small currents $I \ll I_{dep}$ (where contribution of E_{core} to ΔF_V is relatively small) and they concluded that for films with $w > w_c \simeq 4.4\xi$ the energy of single vortex state is lower than the energy of

LA state. Below we show that vortex SP state does not exist at $I \sim I_{dep}$ and it transforms to *vortex free* 2D nucleus (which is a finite region with partially suppressed order parameter) located near the edge of the film. We confirm our above estimation that very near I_{dep} even for wide films $w \gg w_c$ saddle point state of Langer and Ambegaokar has a lowest energy among over SP states. We also study practically important case of the film with one 180^0 bend which models the part of the superconducting meander used in superconducting single photon detectors [19–21]. We show that due to current concentration near the bend the jump to the saddle point state needs much less energy at $I \to I_c$ in comparison with 2D film with uniform current distribution, which promotes the effect of fluctuations in such a samples. This result is rather general and could be applied to any superconducting system with a local current concentration.

II. METHOD

To find the state which is a solution of *stationary* Ginzburg-Landau equations but is *unstable* one (saddle point state) we use the following numerical procedure. In case when we search for SP state *with a vortex* we first put (as an initial condition) the phase distribution corresponding to the vortex seating in the point (n,m) of the discrete grid and additionally fix the phase difference between adjacent points $\varphi(n, m + 1) - \varphi(n, m - 1) = \pi$ (which, in some respect, pins the vortex at the point (n,m)) at any time step. The solution of the stationary GL equations we find by using the relaxation method (by adding the time derivative $\partial\psi/\partial t$ in GL equations and waiting when it goes to zero). By variation of the current (as an external parameter) it is possible to find a stationary state when the vortex does not move from the point (n,m). At low currents there are several points where the vortex position could be fixed by this method for given I. By our definition the one with the highest energy corresponds to SP state (it corresponds to the local maximum of Gibbs energy as function of the vortex position in the London approach [10–12, 14, 15]). After reaching such a state the energy difference could be found using the following expression

$$\Delta F = F_{saddle} - F_{ground} - \frac{\hbar}{2e}I\Delta\varphi \qquad (3)$$

where $\Delta\varphi$ is an additional phase difference between ends of the film which appears in SP state in comparison with the ground state and F_{saddle} and F_{ground} are the Ginzburg-Landau free energies of the saddle point and ground states respectively.

To find the *vortex free* saddle point state we fix the magnitude of the order parameter $|\psi| > 0$ in one point at the edge and allow to vary ψ in all other points. Than we increase the current up to the moment when such a state becomes nonstationary. By our definition we find the vortex free SP state corresponding to *the given value*

of the current. We checked that if we start from this state and let ψ vary everywhere in the film, the vortex is nucleated in the point where we initially fix $|\psi|$ and passes across the film. This finding is an extension to 2D case of the main idea of LA that if one starts from SP state with finite $|\psi|$ everywhere in the sample the time evolution of the order parameter will inevitably lead to the phase slip and voltage pulse.

The proposed method is much simpler (from point of view numerical procedure) than the methods used in Refs. [22, 23] for finding SP states in the mesoscopic superconducting disk in magnetic field or 2D film with a transport current [13, 17]. Similar procedure to fix the vortex position was used, for example, in Ref. [24]. Moreover our method could be easily applied to 2D samples of arbitrary geometry (triangles, disks, stars, etc.) and to 3D case to find the *vortex free* saddle point states. We checked that the method gives the same results for 1D saddle point state (both for the spatial dependence of order parameter and excess energy ΔF) as analytically found in Ref. [6]. Validity of the proposed method is also supported by our results for 2D film because they coincide with results found in the London model at low currents.

In numerical calculations the step of the rectangular grid was $\delta x = \delta y = \xi/4$ and width of the film varied from 4.5 up to 30 ξ. The length of the film was chosen $L = 4w$ (which is long enough to neglect the effect of finite length). The boundary conditions, written here in dimensionless units (for units see for example [23]), were the following: $\psi^*\nabla\psi|_{y=\pm L/2} = iI/wd$ and $\nabla\psi|_{x=\pm w/2} = 0$.

III. RESULTS

In Fig. 1 the dependence of ΔF on the current for different SP states is present for the film with $w = 4.5\xi$. We should note that vortex in such a narrow film has strongly deformed core (see upper inset in Fig. 1) and it resembles more a Josephson vortex than an Abrikosov one [25]. In wider films (see Figs. 2,3) the deformation of a vortex core occurs when the vortex seats near the edge on the distance $\Delta x \lesssim 2\xi$ (see insets in Figs. 2,3). Similar result was found for the vortex placed near the artificial defect (see Fig. 2 in Ref. [24]) and the edge of the superconducting disk (see Fig. 3 in Ref. [22]) or film (see Fig. 2 in Ref. [13]).

Unfortunately our numerical method does not allow to find SP state with a vortex when $\Delta x < 1.5\xi$ (last red circle at the largest current in Figs. (1-3) corresponds to $\Delta x = 1.5\xi$) because we were not able to find the stationary solution of Ginzburg-Landau equation with additional condition $\varphi(n, m + 1) - \varphi(n, m - 1) = \pi$ (this effective pinning 'force' becomes not sufficiently strong to pin the vortex very near the edge). But we notice that if we fix $|\psi| = 0$ along finite length line near the edge (see inset in Fig. 1 where contour plot of $|\psi|$ at
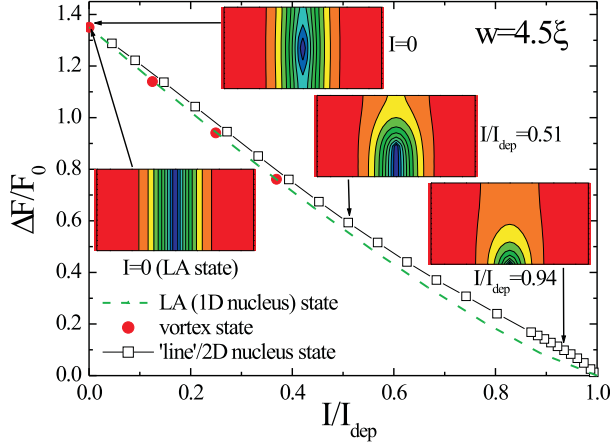
FIG. 1: Energy of saddle point states of three kinds: LA (green dashed curve), vortex/'line' (red circles/rare empty squares) and 2D nucleus (dense empty squares) in the film with $w = 4.5\xi$. In the insets we present the contour plot of $|\psi|$ at different currents and different kinds of SP states.
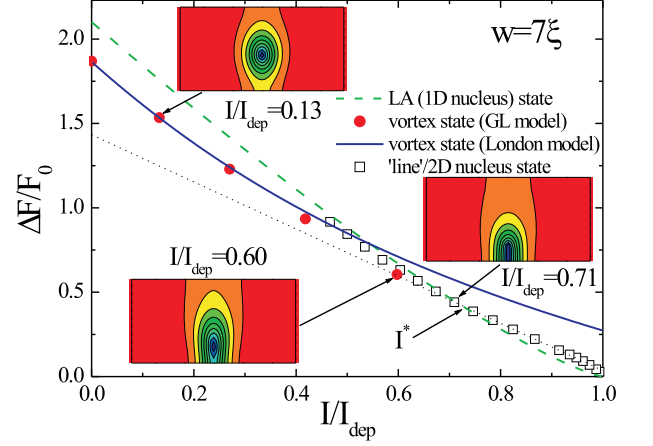

FIG. 2: Energy of saddle point states of three kinds: LA (green dashed curve), vortex/'line' (red circles/rare empty squares) and 2D nucleus (dense empty squares) in the film with $w = 7\xi$. Blue curve corresponds to Eq. (4). Black dotted line is the dependence $\Delta F/F_0 = 1.43(1 - I/1.026I_{dep})$ which fits well (deviation less than 2%) our numerical results at $0.6 \leq I/I_{dep} \leq 0.97$. At $I > I^* \simeq 0.73I_{dep}$ LA state has a lowest energy.

$I/I_{dep} = 0.51$ is shown) and find the stationary solution of GL equations with this additional condition then the excess energy ΔF of such a 'line' state (see rare empty squares in Figs. 1-3) is close to the energy of the vortex state (when vortex seats relatively close to the edge - see Figs. 2,3). Because of that one may approximate the energy of the vortex state when $\Delta x < 1.5\xi$ by the energy of the 'line' state with length $l < 2\xi$.

At currents $I \sim I_{dep}$ the vortex('line') state transforms to the vortex-free SP state (dense empty squares in Figs. 1-3) when the phase circulation along any closed contour in the film is equal to zero and $|\psi| > 0$ everywhere in the film. To find it we fix the amplitude of the order parameter $|\psi|$ in one point at the edge. Because of proximity effect and that $I \sim I_{dep}$ it leads to suppression of $|\psi|$ in the relatively large region around this point (see inset in Fig. 1 at $I/I_{dep} = 0.94$). Further we call it as 2D nucleus state to distinguish it from 1D nucleus state of LA (compare insets in Fig. 1 at $I = 0$ and at $I/I_{dep} = 0.94$).

In Figs. (2,3) we also plot dependence of ΔF for the energy of the vortex SP state found in the London limit [13–15] (solid blue curve)

$$\frac{\Delta F_V}{F_0} = -\frac{1}{2}ln\left(1 + \frac{I^2}{\alpha^2 I_{dep}^2}\right) - \frac{I}{\alpha I_{dep}} \tan^{-1}\left[\frac{\alpha I_{dep}}{I}\right] + $$
$$+\epsilon + ln\left(\frac{2w}{\pi\xi}\right) (4)$$

where $\alpha = 3\sqrt{3}\pi\xi/4w$ and we add the energy of the vortex core $E_{core} = \epsilon F_0$. Numerical coefficient ϵ is found from comparison of Eq. (4) with numerical result at

TABLE I: Values of coefficients in the fitting expressions for energy of vortex/2D nucleus saddle point states (Eq. (5)) and vortex core energy when vortex seats in the center of the film (coefficient ϵ in Eq. (4)).

w/ξ	A	B	C	n	ϵ
7	1.43	1.026	0.89	0.7	0.37
10	1.67	1.026	1.02	0.7	0.38
15	1.85	1.028	0.88	0.6	0.38
30	1.88	1.034	0.68	0.5	0.38

$I = 0$ and it is present in Table 1 for different widths. Notice the good agreement between GL and London model at $I \lesssim 0.6I_{dep}$. At larger currents vortex is located on the distance $\Delta x \lesssim 2\xi$ from the edge and one has to take into account deformation of the core which provides the dependence $\epsilon(I)$. It leads to large discrepancy between London (with $\epsilon = const$) and GL models at $I \gtrsim 0.6I_{dep}$ (see Figs. 2,3). Moreover at $I \sim I_{dep}$ vortex state transforms to the 2D nucleus state which cannot be found in the London limit.

Our numerical results at $I/I_{dep} \gtrsim 0.6$ could be fitted (examples of fitting are present in Fig. 2,3 and inset in Fig. 5) by the following functions

$$\frac{\Delta F}{F_0} \simeq \begin{cases} A(1 - I/BI_{dep}), & 0.6 \leq I/I_{dep} \leq 0.97, \\ C(1 - I/I_{dep})^n, & 0.95 \leq I/I_{dep} \leq 1. \end{cases} (5)$$

Coefficients A,B,C and power n for different widths are listed in Table 1. Note that coefficient A is almost twice

larger than the result which follows from the London model (see Eq. (2)). It reflects the contribution of $E_{core}(I)$ to ΔF which for $I \gtrsim 0.6 I_{dep}$ cannot be neglected and E_{core} gradually decreases with increasing I.
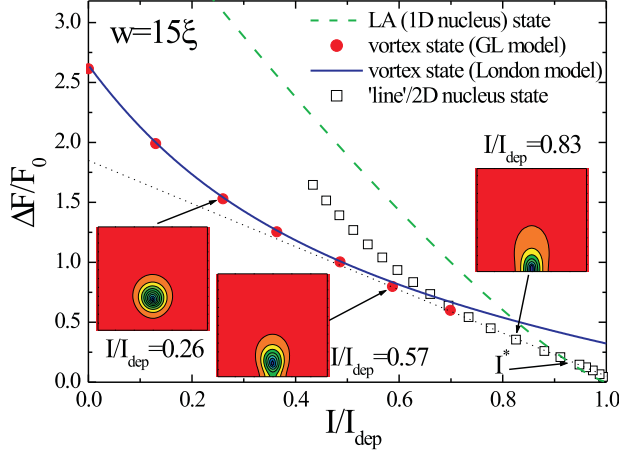

FIG. 3: Energy of saddle point states of three kinds: LA (green dashed curve), vortex/'line' (red circles/rare empty squares) and 2D nucleus (dense empty squares near depairing current) in the film with $w = 15\xi$. Blue curve corresponds to Eq. (4). Black dotted line is the dependence $\Delta F/F_0 = 1.85(1 - I/1.028 I_{dep})$ which fits well (deviation less than 2%) our numerical results at $0.6 \leq I/I_{dep} \leq 0.97$. At $I > I^* \simeq 0.92 I_{dep}$ LA state has a lowest energy.

At $I/I_{dep} \sim 1$ the power $n < 1$ (see Table 1) in Eq. (5) tells one that there is a finite (but rather narrow for wide films with $w \gg \xi$) interval of currents $I^*(w) < I < I_{dep}$ where 1D nucleus (LA) state has the lowest energy (see Figs. 2,3). This counterintuitive, at first sight, result is explained by the presence of the last term in right hand side of Eq. (3). Although in LA state the order parameter is suppressed over whole width (see inset at $I = 0$ in Fig. 1) and it costs more condensation energy than in vortex/2D nucleus state, the phase difference $\Delta\varphi$ is much larger in LA state than in other SP states at $I \sim I_{dep}$ and it brings above result.

Before we consider only single vortex state and 2D nucleus which is located near the edge of the film. In Fig. 4 we demonstrate that the energy of 2D nucleus located in the center of the film is larger than the energy of the edge 2D nucleus. The vortex/antivortex state which is transformed from 2D nucleus state at lower currents (see solid and rare empty squares in Fig. 4) has energy larger than the single vortex state. The similar result was found in London limit where the difference reaches two times between ΔF_V and ΔF_{VA} [15, 17].

We also find the saddle point states in 2D superconducting film with a 180^0 bend - see left inset in Fig. 5. In our simulations we choose the width of the film $w = 10\xi$,
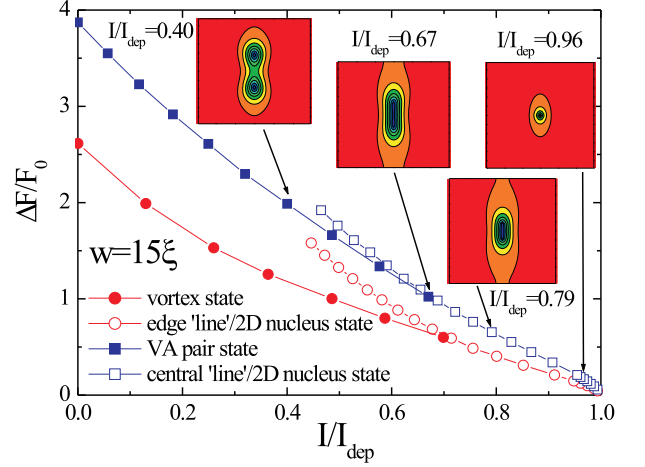

FIG. 4: Energy of saddle point states of different kinds: single vortex (red circles), vortex-antivortex pair (blue squares) and 'line'/2D nucleus located at the edge (empty rare/dense circles) and in the center (empty rare/dense squares) of the film.

the length of the sample in the bend region L=2w and two widths of the slit: $w_{slit} = 2.5\xi$ (shown in the left inset in Fig. 5) and $w_{slit} = 10\xi$. In Fig. 5 we present our results for the energy of single vortex and edge 2D nucleus states. Note that the current in Fig. 5 is normalized to the critical current of the sample and not to the I_{dep} as in Figs. 1-4 ($I_c = 0.85 I_{dep}$ for film with $w_{slit} = 2.5\xi$, $I_c = 0.91 I_{dep}$ for film with $w_{slit} = 10\xi$ and $I_c = I_{dep}$ for straight film).

We want to stress here that at $I \sim I_c$ the energy of SP states is considerably lower in the film with bend than without it (compare results present in right inset in Fig. 5) taken at *the same ratio* I/I_c. And the stronger the current concentration near the bend (which is manifested in lower value of I_c for smaller w_{slit}) the smaller ΔF. We expect that the smallest ΔF could be reached in case of infinitesimally narrow and long crack near the edge of the film [26, 27] which provides the maximal current concentration and maximal suppression of I_c. We explain this effect by the partial suppression of the order parameter (on the scale of about several ξ) in the region with the strongest current concentration even in the ground state. As a result it takes less energy to jump to SP state from the ground state with already locally suppressed $|\psi|$. The proof of this idea also comes from our results at low currents where ΔF differs a little for straight film and film with a bend (see Fig. 5). At low currents the suppression of $|\psi|$ near the bend is weak and it slightly influences ΔF.

The decrease of ΔF in the 1D wire with constriction was also found in Ref. [28]. But authors of [28] did not present ΔF as function of I/I_c and it is not clear is their

result connected with reduction of I_c (which gives trivial $\Delta F \to 0$ at current $I = I_c < I_{dep}$ and $\Delta F < \Delta F_{LA}$ at $I < I_c$) or with a change in the power n in asymptotics $\Delta F \sim (1 - I/I_c)^n$ as in our case (see right inset in Fig. 5).
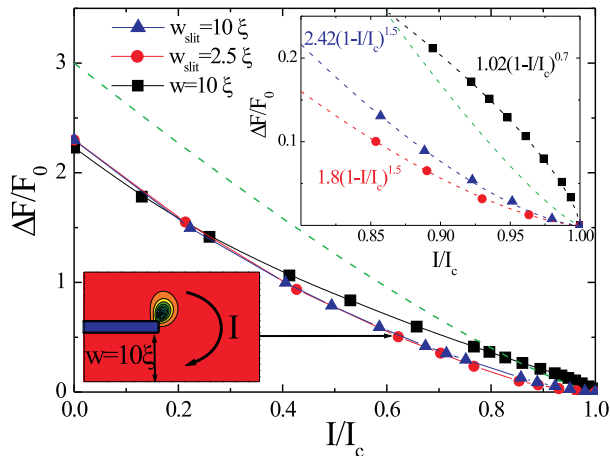

FIG. 5: Energy of saddle point states (vortex and 2D nucleus) for the film with a bend and slits of different width: $w_{slit} = 2.5\xi$ (red circles) and $w_{slit} = 10\xi$ (blue triangles) and the film without bend with $w = 10\xi$ (black squares). In the right inset we present the zoom at $I \sim I_c$ which shows the energy of 2D nucleus SP states with dashed curves as the fitting of the numerical results. Green dashed curve shows the energy of LA state for straight film with $w = 10\xi$.

for wide films $w \gg \xi$) the lowest energy SP state corresponds either to edge 2D nucleus state or to state with a vortex located next to the edge and having strongly modified core. At currents $I \lesssim 0.6 I_{dep}$ (for films with $w \gtrsim 7\xi$) the lowest energy saddle point state corresponds to the single vortex with ordinary core and the results of the London model are recovered. We demonstrate that in the film with current concentration, which leads to the local spatial variation of the order parameter in the ground state, the energy of 2D nucleus SP state may be much lower than the energy of any saddle point states in the film with uniform current distribution taken at the same ratio $I/I_c \sim 1$.

The last result has an important practical consequence. It shows that if in the sample there are places with strong current concentration (bends, geometrical defects of the edge and so on) it favors the effect of fluctuations near I_c because of much low, in comparison with straight film without defects, energy barrier. For example, if $k_B T = 0.1 F_0$ the switching of the straight film with $w = 10\xi$ to resistive state roughly occurs at $I \simeq 0.96 I_c = 0.96 I_{dep}$ (at this current $\Delta F = 0.1 F_0$ - see right inset in Fig. 5). But for the film with a bend and $w_{slit} = 2.5\xi$ the energy barrier $\Delta F = 0.1 F_0$ is reached at $I \simeq 0.85 I_c \simeq 0.72 I_{dep}$ and transition to resistive state occurs at much lower current than one could expect from results for straight film.

We believe that the found results are valid not only near T_c (where Ginzburg-Landau model is quantitatively valid) but at lower temperatures too. Indeed, in the recent work [29] asymptotics $\Delta F \sim (1 - I/I_{dep})^{5/4}$ for $I \to I_{dep}$ was confirmed for LA like state in 'dirty' 1D superconducting wire at any temperature. In the 'pure' 1D wire only 15% difference with LA result at $T \to 0$ was noticed in Ref. [30] in the limit $I \to 0$ [31].

IV. CONCLUSION

It is found that in 2D superconducting film with spatially uniform current distribution in the ground state the lowest energy saddle point state at $I \sim I_{dep}$ corresponds to the state of Langer and Ambegaokar found for 1D wire. Only at $I < I^*$ (where $I^*(w) < I_{dep}$ even

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